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## D-BRANE WESS-ZUMINO ACTIONS, T-DUALITY AND THE COSMOLOGICAL CONSTANT

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A geometrical formulation of the T-duality rules for type II superstring Ramond–Ramond fields is presented. This is used to derive the Wess-Zumino terms in superstring D-brane actions, including terms proportional to the mass parameter of the IIA theory, thereby completing partial results in the literature. For non-abelian world-volume gauge groups the massive type IIA D-brane actions contain non-abelian Chern–Simons terms for the Born–Infeld gauge potential, implying a quantization of the IIA cosmological constant.

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# 1. Introduction

The bosonic sector of the effective world-volume action for a type IIA superstring Dirichlet  $p$ -brane in a bosonic type II supergravity background takes the form

$$I = I_{DBI} + I_{WZ}. \quad (1.1)$$

The first term is the Dirac–Born–Infeld (DBI) action [1-9],

$$I_{DBI} = -T_p \int d^{p+1}\xi \, e^{-\phi} \sqrt{-\det \left( g_{ij} - B_{ij} + \frac{\alpha'}{2\pi} F_{ij} \right)}, \quad (1.2)$$

where  $\phi$ ,  $g_{ij}$  and  $B_{ij}$  are the pullbacks to the world-volume of the Neveu–Schwarz–Neveu–Schwarz ( $\text{NS} \otimes \text{NS}$ ) supergravity fields and  $F = dV$ , where  $V$  is the Born–Infeld 1-form  $U(1)$  gauge field. The constant  $T_p$  is the  $p$ -volume tension with mass dimension  $p + 1$ . The second term in (1.1) is the ‘Wess–Zumino’ (WZ) term describing the coupling of the  $D$ -brane to the background Ramond–Ramond ( $\text{R} \otimes \text{R}$ ) fields. We can assemble these fields into the complex of differential forms,

$$C = \sum_{r=0}^9 C^{(r)}, \quad (1.3)$$

where  $C^{(r)}$  is a differential form of degree  $r$ . The fields  $C^{(r)}$  are the  $\text{R} \otimes \text{R}$  gauge potentials of either IIA ( $r$  odd) or IIB ( $r$  even) supergravity. The 9-form potential is optional because its equations of motion force the dual of its field strength to be a constant,  $m$  [10-12]. One can set  $m = 0$  but  $m \neq 0$  is also possible, in which case the background fields are those of massive IIA supergravity [13].

When  $C^{(9)} = 0$ , i.e.  $m = 0$ , one has [5],

$$I_{WZ}^{(p+1)} = T_p \int_{W_{p+1}} C e^{\left(\frac{\alpha'}{2\pi} F - B\right)}, \quad (1.4)$$

where it is to be understood that one selects the  $(p + 1)$ -form in the expansion of the integral and that all forms in space-time are pulled back to the  $(p + 1)$ -dimensional world-volume,  $W_{p+1}$ . One purpose of this paper is to generalize (1.4) to the  $m \neq 0$  case, thereby generalizing previous partial results along these lines [14].

It is convenient to define

$$\hat{F} = F - \frac{2\pi}{\alpha'} B, \quad (1.5)$$

where  $B$  is again the pullback to the world-volume of the corresponding space-time two-form. In the generalization from a  $U(1)$  gauge field to a  $U(n)$  gauge field, which is appropriate for  $n$  coincident  $D$ -branes [3], the DBI world-volume 1-form gauge potential  $V$  takes values in the Lie algebra of  $U(n)$  and its Lie-algebra valued field strength  $F$  is given by

$$F = dV + V^2. \quad (1.6)$$

In this case we can still define  $\hat{F}$  as in (1.5) if  $B$  is assumed to take values in the Lie algebra of  $U(1) \subset U(n)$ .

The WZ term (1.4) has the following straightforward generalization to the  $U(n)$  case [5],

$$I_{WZ}^{(p+1)} = T_p \int_{W_{p+1}} C \operatorname{tr} \left( e^{\alpha' \hat{F}/2\pi} \right), \quad (1.7)$$

where the trace is taken in the  $n$ -dimensional representation of  $U(n)$ . The DBI term will require more extensive modification, but in this paper we shall be concerned exclusively with the WZ term.

It should be noted that  $C$  includes both the standard  $R \otimes R$  potentials of IIA and IIB supergravity in its form expansion *and* their duals. It is consistent to introduce both a potential and its dual if the background fields are on shell because the Bianchi identities and the field equations are then on the same footing. The  $C^{(9)}$  gauge potential is exceptional in a number of respects. For example, it has no dual potential. Its field strength does have a dual field strength but the field equations restrict this to be a constant,  $m$ . This constant, which has dimensions of mass, is essentially the square root of the cosmological constant appearing in the ‘massive’ IIA supergravity theory [13]. Non-zero  $C^{(9)}$  is therefore equivalent to non-zero  $m$  [10-12]. In such a background the WZ term (1.7) requires  $m$ -dependent modifications.

These modifications have been found for  $p = 0, 2$  in the  $U(1)$  case [14]. One result of this paper will be to extend these results to all even  $p$  and to gauge groups  $U(n)$ .

Specifically, we will show that when  $m \neq 0$  the integrand of (1.7) is replaced (up to a total derivative) by

$$L = C \text{tr} \left( e^{\alpha' \hat{F}/2\pi} \right) + m \sum_{r=0} \frac{1}{(r+1)!} \omega_{2r+1}^{(0)}, \quad (1.8)$$

where  $\omega_{2r+1}^{(0)}$  is a Chern-Simons  $(2r+1)$ -form with the property that

$$d\omega_{2r+1}^{(0)} = \text{tr} \left( \frac{\alpha' F}{2\pi} \right)^{r+1}. \quad (1.9)$$

The form expansion of  $L$  gives the WZ lagrangians used to construct  $I_{WZ}^{(p+1)}$ .

Since the  $m$ -dependent term in (1.8) contributes only odd forms in the form expansion of  $L$ , only the  $D$ -brane actions for *even*  $p$  (i.e. those of the type IIA theory) acquire  $m$ -dependent corrections. This is expected because the constant  $m$  can be considered as a  $\mathbb{R} \otimes \mathbb{R}$  field of the type IIA theory, equivalent to  $C^{(9)}$ . One way to see that these  $m$ -dependent terms must be present in the IIA  $D$ -brane actions is by T-duality with the type IIB  $D$ -brane actions. This was shown for  $p \leq 3$  and with a  $U(1)$  gauge group in [14] using the ‘massive’ T-duality rules of [12] (which complement the ‘massless’ rules given in [15]). We will generalize this procedure to establish that the  $m$ -dependent term in (1.8) is precisely that required by T-duality. This result also completes previous results [16–18] on T-duality of  $D$ -brane actions for  $m = 0$ . However, since the only known supersymmetric backgrounds that solve the IIA field equations require the presence of an eight-brane the  $D$ -brane actions for  $m \neq 0$  should be interpreted as describing the dynamics of the  $D$ -brane in the presence of an eight-brane. In such a background there can be additional world-volume fields arising from open strings connecting the  $D$ -brane to the eight-brane. These will not be taken into account in this paper.

With a non-abelian  $U(n)$  gauge group the  $m$ -dependent terms in (1.8) are non-abelian Chern-Simons (CS) forms. As described in a different context in [19,20], quantum consistency in such cases requires a quantization of the CS coefficient which, in this case, is the mass parameter  $m$ . Thus, an important consequence of the results of this paper is the quantization of the IIA superstring cosmological constant, as argued previously [11,12] for quite different reasons. This will be discussed further at the end of this article.

## 2. D-brane actions in $R \otimes R$ backgrounds

In order to explain the form of the D-brane WZ actions it will prove very useful to first provide a uniform formulation of the  $R \otimes R$  and  $NS \otimes NS$  gauge symmetries. In this and the next section we shall set  $\alpha' = 1$  for convenience.

The differential forms of even degree appearing in (1.3) are the  $R \otimes R$  gauge potentials of IIB supergravity while those of odd degree are the  $R \otimes R$  potentials of IIA supergravity. These  $R \otimes R$  fields are subject to the gauge transformation,

$$\delta_\Lambda C = d\Lambda - H \wedge \Lambda, \quad (2.1)$$

where  $H = dB$  is the  $NS \otimes NS$  3-form field strength and

$$\Lambda = \sum_{r=0}^8 \Lambda^{(r)}. \quad (2.2)$$

A further  $R \otimes R$  transformation is

$$\delta_\lambda C = \lambda e^B, \quad (2.3)$$

for *constant*  $\lambda$ . It will be convenient for our purposes to combine the  $R \otimes R$  transformations into

$$\delta_{RR} C = d\Lambda - H \wedge \Lambda + \lambda e^B. \quad (2.4)$$

These transformations encapsulate the local  $R \otimes R$  symmetry of the IIA and IIB supergravity lagrangians. The specific choice of field variables, and hence of the transformations, has been chosen in order to clarify the invariances of the D-brane WZ actions. Specifically,

$$\delta_{RR} L = d \left[ \Lambda \text{tr} \left( e^{\hat{F}/2\pi} \right) + \lambda \sum_{r=0} \frac{1}{(r+1)!} \omega_{2r+1}^{(0)} \right], \quad (2.5)$$

which means that the WZ action (1.7) changes by a surface term. An intriguing feature of this result is that

$$\delta_{RR} L|_{\Lambda=C, \lambda=m} = dL, \quad (2.6)$$

so that the  $(p+1)$ -form lagrangians in the expansion of  $L$  are related by a type of cohomological descent.

We also need to consider the transformation of the background  $R \otimes R$  fields under the  $NS \otimes NS$  gauge transformation  $\delta_\eta B = d\eta$ . When  $m = 0$  the  $R \otimes R$  potentials are invariant,  $\delta_\eta C = 0$ , but when  $m \neq 0$  they transform non-trivially [12]. In terms of the field definitions in this paper these transformations are,

$$\delta_\eta B = d\eta, \quad \delta_\eta C = -me^B \eta. \quad (2.7)$$

It is clear from the form of the WZ lagrangian when  $m = 0$  that it is invariant under the  $\eta$  gauge transformation provided that

$$\delta_\eta V = 2\pi\eta, \quad (2.8)$$

where  $\eta$  is considered to take values in the Lie algebra  $U(1) \subset U(n)$ . The D-brane lagrangian for  $m \neq 0$  can be found from the requirement of  $\eta$  invariance as we shall see later. Note that

$$[\delta_\eta, \delta_\lambda] = \delta_\Lambda|_{\Lambda=\lambda\eta e^B}, \quad (2.9)$$

with all the other commutators vanishing, so that the combined  $\eta, \lambda, \Lambda$  transformations form a closed algebra.

The  $NS \otimes NS$  field strength  $H = dB$  is obviously invariant under all the above transformations. So is the field strength of the  $R \otimes R$  fields,

$$R(C) = dC - H \wedge C + me^B. \quad (2.10)$$

The form expansion of  $R(C)$  yields all the ‘modified’ field strengths of the IIA and IIB potentials *and* their duals. To relate the potentials to their duals we first define

$$R_{<} = \sum_{r=0}^5 R^{(r)}, \quad R_{>} = \sum_{r=5}^{10} R^{(r)}, \quad (2.11)$$

and then impose the constraint,

$$R_{>} = *R_{<}, \quad (2.12)$$

where  $*$  is the Hodge dual in ten dimensions. This relates the potentials of the background supergravity theory to their duals and also imposes self-duality of the IIB five-form field strength. Also, note that (2.12) implies that the ten-form field strength is

such that

$$*R^{(10)} = m, \quad (2.13)$$

which is appropriate when  $m \neq 0$ .<sup>★</sup> The Bianchi identity takes the form

$$dR(C) - H \wedge R = d \left[ R(C) e^{-B} \right] = 0. \quad (2.14)$$

The  $m = 0$  WZ action (1.7) can be expressed as an integral of a covariant expression in  $p + 2$  dimensions on a manifold  $M_{p+2}$  with boundary  $W_{p+1}$ ,

$$I_{WZ}^{(p+1)} = T_p \int_{M_{p+2}} R(C) \text{tr} \left( e^{\hat{F}/2\pi} \right). \quad (2.15)$$

It is natural to suppose that this action remains valid when  $m \neq 0$  because there is no other candidate integrand that is both gauge invariant and reduces to the one known to be correct when  $m = 0$ . When  $m \neq 0$  the integrand of (2.15) can be written as  $dL$  where  $L$  is as given in (1.8). As the addition of any closed form to  $L$  yields the same expression for  $dL$  there is an intrinsic ambiguity. We will find it convenient to use the alternative, but equivalent, lagrangian

$$L_{WZ} = [nC + \omega R(C)] e^{-B}, \quad (2.16)$$

where  $\omega$  is defined by the requirement that

$$d\omega = \text{tr} \left( e^{F/2\pi} - 1 \right). \quad (2.17)$$

It is obvious from the construction that the lagrangian (2.16) is  $\eta$ -invariant up to a total derivative, and a calculation shows that  $\delta_\eta L_{WZ} = -d(\eta C e^{-B})$ . Also,  $L_{WZ}$  varies by a total derivative under the gauge transformation  $\delta_\chi V = D\chi$ . The action is, however, not necessarily invariant under ‘large’ gauge transformations, as will be discussed in the last section of this paper.

We shall now show that the WZ lagrangians contained in the form expansion of (2.16) are related by T-duality.

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★ The corresponding relation in [12] looks more complicated when  $B \neq 0$  but must be equivalent after field redefinitions. This suggests that the field definitions used here might significantly simplify the massive IIA supergravity.

### 3. T-duality of D-brane WZ actions

T-duality of Dirichlet  $p$ -brane actions amounts to the statement that the double dimensional reduction of the  $p$ -brane action yields the ‘direct’ reduction of the  $(p - 1)$ -brane action if the background fields of the two actions are related by T-duality. We shall therefore begin by discussing T-duality for the background fields.

T-duality presupposes the existence of a  $U(1)$  symmetry of the background. We can choose coordinates such that the associated Killing vector field is  $\partial/\partial y$ . In this case, the metric, dilaton and all field strengths are independent of  $y$ . In addition, we will make the simplifying assumption that

$$i_y H = 0, \quad (3.1)$$

where  $i_y$  ( $\equiv *dy*$ ) indicates contraction with  $\frac{\partial}{\partial y}$ . With this assumption  $H$  is invariant under T-duality, as can be seen from the T-duality rules of [15], and this greatly simplifies the discussion of the T-duality transformation of the WZ term.

The T-duality transformations of the  $R \otimes R$  field strengths (implicit in the T-duality rules of [15,12]) are given by

$$R(C) \rightarrow dy \wedge R(C) + i_y R(C), \quad (3.2)$$

where  $\rightarrow$  means that the fields appearing on the left are replaced by those on the right. Since  $(dy + i_y)^2 = 1$ , T-duality is an invertible  $Z_2$  transformation between the IIA and IIB fields. The map (3.2) of the field strengths is induced by

$$C \rightarrow -(dy \wedge C + i_y C), \quad (3.3)$$

provided that

$$\mathcal{L}_y C = (m - mdy)e^B, \quad (3.4)$$

where  $\mathcal{L}_y$  is the Lie derivative with respect to  $\partial/\partial y$ . This implies, in particular, that

$$C^{(0)} \leftrightarrow -C_y^{(1)}, \quad (3.5)$$



and (3.4) determines  $C^{(0)}$  and  $C_y^{(1)}$  to be of the form,

$$C^{(0)} = my + \bar{C}^{(0)}, \quad C_y^{(1)} = -my + \bar{C}_y^{(1)}, \quad (3.6)$$

where  $\bar{C}^{(0)}$  and  $\bar{C}_y^{(1)}$  are independent of  $y$ , in agreement with [12]. Despite the  $y$ -dependence of the potential  $C$  when  $m \neq 0$ , the field strength  $R(C)$  is  $y$ -independent since

$$\mathcal{L}_y R(C) = 0. \quad (3.7)$$

We shall now compare the double-dimensional reduction of the  $p$ -brane WZ action with the direct reduction of the  $(p-1)$ -brane WZ action. The double-dimensional reduction proceeds as follows. We write

$$L_{WZ} = L_{WZ}^+ + L_{WZ}^-, \quad (3.8)$$

where  $L_{WZ}^\pm$  are the projections of  $L_{WZ}$  defined by

$$L^+ = d\sigma \wedge i_\sigma L, \quad L^- = i_\sigma(d\sigma \wedge L), \quad (3.9)$$

for arbitrary world-volume form  $L$ , where  $\sigma$  is a particular world-volume coordinate. The  $(p+1)$ -dimensional world-volume  $W_{p+1}$  will be taken to be of the form  $W_p \times S^1$ , where  $\sigma$  is the  $S^1$  coordinate. If the  $p$ -brane is now wrapped around the  $S^1$  factor of space-time, with coordinate  $y$ , then the two  $S^1$  coordinates can be identified,

$$dy = d\sigma. \quad (3.10)$$

Moreover, only the  $L_{WZ}^+$  projection of  $L_{WZ}$  contributes to the integral over  $W_{p+1}$ . We shall now work towards a convenient expression for  $L_{WZ}^+$ .

We first observe that the world-volume one-form gauge potential  $V$  can be written as

$$V = d\sigma V_\sigma + V^-, \quad (3.11)$$

and hence

$$F = -d\sigma \wedge DV_\sigma + F^-, \quad (3.12)$$

where  $DV_\sigma = dV_\sigma + [V, V_\sigma]$ . To establish the T-duality map between the D-brane actions in the non-abelian case ( $n > 1$ ) it will be necessary to restrict  $V_\sigma$  to the Lie algebra of

the  $U(1)$  subgroup of  $U(n)$ . We therefore set

$$V_\sigma = -2\pi\Phi, \quad (3.13)$$

where  $\Phi$  is a scalar world-volume field. If we take the compactification radius to be  $l$  so that  $\sigma$  is identified with  $\sigma + 2\pi l$  (using  $dy = d\sigma$ ), then a  $U(1)$  transformation of  $V$  with group element  $e^{i\sigma/l}$  shifts  $\Phi$  to  $\Phi + 2\pi\tilde{l}$ , where  $\tilde{l} = 4\pi^2$ . The world-volume field  $\Phi$  is therefore a map from the  $S^1$  factor of space-time in the T-dual theory to a  $(p-1)$ -brane world-volume. Using (3.13) we have  $DV_\sigma = -2\pi d\Phi$ , and hence, from (3.12)

$$\text{tr} \left( e^{F/2\pi} \right) = \left[ \text{tr} \left( e^{F/2\pi} \right) \right]^- + d\sigma \wedge d\Phi \left( e^{F/2\pi} \right), \quad (3.14)$$

from which it follows that

$$\omega^+ = d\sigma \wedge d\Phi \wedge \omega - nd\sigma \wedge \Phi. \quad (3.15)$$

Space-time forms can be decomposed in the same manner as (3.8), so that,

$$R(C) = R^+(C) + R^-(C), \quad (3.16)$$

where

$$R^+(C) = dy \wedge i_y R(C), \quad R^-(C) = i_y(dy \wedge R(C)). \quad (3.17)$$

In view of the relation  $dy = d\sigma$ , the  $\pm$  components of space-time forms pull back to  $\pm$  world-volume forms. Noting also that  $H = H^-$  by virtue of the assumption (3.1), we have

$$L_{WZ}^+ = C^+ n e^{-B} + \omega R^+ e^{-B} + \omega^+ R^- e^{-B}. \quad (3.18)$$

Substituting the expression (3.15) into the last term gives

$$\omega^+ R^-(C) e^{-B} = -d\sigma \wedge \left[ \omega d\Phi R^- e^{-B} - nd\Phi C^- e^{-B} \right] - nd\sigma \wedge d \left[ \Phi C e^{-B} \right]. \quad (3.19)$$

The last term in this expression will give a surface term in the integral over  $W_p$  and can

be ignored. Thus,

$$\int_{W_{p+1}} L_{WZ} = \int_{W_{p+1}} L_{WZ}^+ = \int_{W_p \times S^1} [n(C^+ + d\sigma d\Phi C^-) + (\omega R^+ - d\sigma \omega d\Phi R^-)] e^{-B}. \quad (3.20)$$

We could now perform the  $\sigma$  integral to get the double dimensional reduction of  $L_{WZ}$ , which would then have to be rewritten using the T-duality rules. It is more convenient to reverse the order of these steps, making use of the fact that the T-duality transformations of the components of  $C$  and  $R(C)$  are,

$$\begin{aligned} C^+ &\rightarrow -dy \wedge C^+, & C^- &\rightarrow -i_y C^-, \\ R^+(C) &\rightarrow dy \wedge R(C), & R^-(C) &\rightarrow i_y R(C). \end{aligned} \quad (3.21)$$

This yields, on setting  $dy = d\sigma$ ,

$$\int_{W_p \times S^1} L_{WZ} \rightarrow - \int_{S^1} d\sigma \int_{W_p} [n(C^- + d\Phi i_y C) + \omega^-(R^- + d\Phi i_y R)] e^{-B}. \quad (3.22)$$

Before attempting to integrate over  $\sigma$  we should determine whether the integrand is  $\sigma$ -dependent. This possibility arises when  $m \neq 0$  because in that case  $C$  is  $y$ -dependent. In fact, using (3.4),

$$C e^{-B} = \bar{C} e^{-B} + m y - m y d y, \quad (3.23)$$

where  $\bar{C}$  is  $y$ -independent. Thus,

$$(C^- + d\Phi i_y C) e^{-B} = (\bar{C}^- + d\Phi i_y \bar{C}) e^{-B} + m y - m y d\Phi. \quad (3.24)$$

The  $m y$  term contributes a constant to the zero-brane lagrangian, and may be ignored, while the  $m y d\Phi$  term contributes a total derivative (proportional to  $m d\Phi$ ) to the one-brane lagrangian, which may also be ignored. Effectively, therefore, the  $W_p$  integral in (3.22) is  $\sigma$ -independent so performing the  $\sigma$  integral yields

$$\int_{W_p} L_{WZ} \rightarrow -2\pi l \int_{W_p} [n(C^- + d\Phi i_y C) + \omega^-(R^- + d\Phi i_y R)] e^{-B}. \quad (3.25)$$

If  $\Phi$  is now interpreted as the  $S^1$  space-time coordinate then a form such as  $C^- + d\Phi i_y C$  is just the decomposition into  $\mp$  projections of the  $D = 10$  form  $C$ , i.e. it is the ‘direct’

reduction of  $C$ . Furthermore, the form  $\omega^-$  on  $W_{p+1}$  is just  $\omega$  on  $W_p$  since  $W_p$  has no  $\sigma$  coordinate. We conclude that the integral on the right-hand side of (3.25) is just  $L_{WZ}$ , so

$$T_p \int_{W_{p+1}} L_{WZ} \rightarrow -2\pi l T_p \int_{W_p} L_{WZ}. \quad (3.26)$$

This result shows that the D-brane actions are related by T-duality provided that<sup>★</sup>

$$T_{p-1} = (2\pi l_p) T_p, \quad (3.27)$$

where  $l_p$  is the radius of the compact dimension in the  $D = 10$  theory with the Dirichlet  $p$ -brane; that is,  $l_p = l_A$  for  $p$  even and  $l_p = l_B$  for  $p$  odd, where  $l_A$  and  $l_B$  are related by  $l_A l_B = 4\pi^2$ . It should be remarked that had we used (1.8) instead of (2.16) then it would have been necessary to consider boundary terms in the action to establish T-duality. This accounts for the ambiguity in the approach of [14] in which, in effect, boundary terms are ignored.

For later considerations it will be important to appreciate that the constants  $T_p$  are not the physical tensions; these are

$$T_p^{phys} = \frac{T_p}{g}, \quad (3.28)$$

where  $g$  is the string coupling constant. Under  $T$ -duality  $g \rightarrow 2\pi l g$ , so the physical tension of the  $(p-1)$ -brane found by double dimensional reduction is

$$T_{p-1}^{phys} = T_p^{phys}. \quad (3.29)$$

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★ The sign is irrelevant since the tensions should be identified with the absolute values of the WZ coefficients.

#### 4. Quantization of the cosmological constant

When  $m \neq 0$  the WZ lagrangian for even  $p$  contains the term

$$I_p(V) = mT_{2r} \frac{1}{(r+1)!} \int_{W_{2r+1}} \omega_{2r+1}^{(0)}, \quad (p = 2r), \quad (4.1)$$

which is a Chern-Simons term for the *world-volume*  $U(n)$  gauge field. As is well known, under a gauge transformation  $V \rightarrow g^{-1}Vg + g^{-1}dg$  the action (4.1) changes by a term proportional to the winding number of  $g(\xi)$ . If  $W_{p+1}$  is the  $(p+1)$ -sphere then the maps  $g$  are classified by  $\pi_{p+1}(G)$ , for a gauge group  $G$ . For  $G = U(n)$  one has  $\pi_{2p+1} \supset \mathbb{Z}$ . Thus, for sufficiently large  $n$  there are always large  $U(n)$  gauge transformations for which the action is not invariant. As shown in [19] single valuedness of  $e^{iI_p(V)}$ , required for quantum consistency of the world-volume field theory, implies a quantization of the coefficient of  $I_p(V)$ . The resulting quantization condition in our case is

$$mT_p(\alpha')^{1+p/2} = 2\pi\nu, \quad (p \text{ even}), \quad (4.2)$$

for integer  $\nu$ . This is actually a series of quantization conditions, one for each even value of  $p \leq 8$ . The consistency of these relations requires (for  $p$  even and a given  $\nu$ )

$$\alpha' T_p = T_{p-2}, \quad (4.3)$$

which can be shown to be satisfied (for any  $p$ ) by iteration of (3.27) and use of the relation

$$l_A l_B = 4\pi^2 \alpha'. \quad (4.4)$$

The relations (4.3) should not be confused with the well-known quantization condition on the products  $T_p T_{6-p}$ .

The physical tensions  $T_p^{phys} = T_p/g$  should be independent of the radius. Since the coupling  $g$  is not invariant under T-duality we shall set  $g = g_A$  or  $g = g_B$ , according to which of the two type II theories we are considering. Consider first the IIB theory. The

condition that  $T_p^{phys}$  is independent of the compactification radius implies that

$$T_p = cg_B(\alpha')^{-(p+1)/2}, \quad (p \text{ odd}), \quad (4.5)$$

for some dimensionless constant  $c$  that is independent of  $g_B$  (and is determined by normalization conventions in the string theory). One can now use this in (3.27) to obtain an expression for  $T_p$  when  $p$  is even. Substitution of this into (4.2) gives

$$m = (cg_B l_B)^{-1} \nu. \quad (4.6)$$

If  $g_B$  is set to unity the quantization condition  $m \propto \nu/l_B$  of [12] is recovered. The result in [12] was obtained by requiring consistency between T-duality and  $SL(2; Z)$  U-duality of type IIB theory. However, the factor of  $g_B^{-1}$  is important for the purpose of rewriting (4.6) in IIA terms since  $g_B l_B = 2\pi\sqrt{\alpha'} g_A$ . Thus, the mass quantization condition may be written entirely in terms of the IIA theory as

$$m = \frac{1}{2\pi c g_A \sqrt{\alpha'}} \nu. \quad (4.7)$$

The same result follows more directly from the condition that the physical D-brane tensions of the IIA theory be independent of  $l_A$ . Thus the mass scale set by the quantization condition of  $m$  is not a new scale in the theory but is the same as the one set by the zero-brane mass.

We will conclude with some comments on the  $p = 2$  case, which is of particular interest since it describes a membrane that is supposed to descend from the membrane of eleven-dimensional M-theory [4,6]. Let

$$V = V_0 + \tilde{V}, \quad (4.8)$$

where  $V_0$  is a  $U(1)$  gauge potential and  $\tilde{V}$  an  $SU(n)$  gauge potential. Then  $I_p(V)$  can be written as<sup>★</sup>

$$\frac{mT_2}{2} \int_{W_3} V_0 dV_0 + \frac{mT_2}{2} \int_{W_3} \text{tr} \left( \tilde{V} d\tilde{V} + \frac{2}{3} \tilde{V}^3 \right). \quad (4.9)$$

The first term is the topological mass term found in [14]. As pointed out in [14] this term prevents the dualization of  $V_0$  to a world-volume scalar and hence appears to obstruct

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★ A CS term for the euclidean  $p = 2$  D-brane has appeared previously in a different context [21].

the  $D = 11$  interpretation of the Dirichlet two-brane. Indeed, the mass of the type IIA theory has no known 11-dimensional interpretation, and this is an interesting challenge to the idea that all superstring theories should be unified in  $D = 11$  M-theory. The quantization condition (4.7) might help resolve this puzzle since  $m$  is quantized in units of  $1/R_{11}$ , where  $R_{11}$  is the radius of the eleventh dimension. This unit therefore goes to zero in the decompactification limit.

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